AIAA JOURNAL
Vol. 31, No. 10, October 1993

Application of Lamination Parameters to Reliability-Based Stiffness Design of Composites

Mitsunori Miki,* Yoshisada Murotsu,† and Nobuhiko Murayama‡
University of Osaka Prefecture, Sakai 593, Japan
and
Tetsuo Tanaka§
Matsushita Electric Works, Ltd., Kadoma, Osaka 571, Japan

This paper is concerned with the optimum stiffness design of multiaxial fiber reinforced laminates under the probabilistic conditions of loads and material properties using the lamination parameters. The constraints are imposed on the in-plane stiffnesses or their functions with a prescribed reliability. The reliability analysis of the stiffness and the strains of the laminates under the probabilistic in-plane stresses is conducted by using the advanced first-order second-moment (AFOSM) method. Then, the optimum fiber orientation angles of multiaxial laminates are determined based on reliability and compared with those under the deterministic conditions. The effects of the probabilistic properties of the applied loads and the elastic constants of the ply material are discussed from the viewpoint of reliability and optimum design.

Nomenclature

A = in-plane stiffness matrix
A^* = normalized in-plane stiffness matrix
A^*_{ij} = element of normalized in-plane stiffness matrix, $i, j = 1, 2, 6$
CC = correlation coefficient
CV = coefficient of variation
$CV(X)$ = coefficient of variation of X
D = flexural stiffness matrix
D^* = normalized flexural stiffness matrix
D^*_{ij} = element of normalized flexural stiffness matrix, $i, j = 1, 2, 6$
$E(X)$ = mean value of X
E_i = effective in-plane engineering constant of elasticity, $i = 1, 2, 6$
F_X = probability distribution function of X
f = objective function
f_X = probability density function of X
g, g_k = constraint
h = thickness of laminated plate
K = number of constraints; failure constant of laminate
M = moment vector
$M, M(x)$ = limit state function
N = in-plane load vector
N = half-number of total plies
N_i = number of plies in the ith ply group
$P[X]$ = probability of X
P_f = probability of failure
P_0 = lower bound of probability
p_i = ply ratio of the ith ply group
SD = standard deviation
S_i = stress, $i = 1, 2, 6$
SV = sum of variances
U, u = transformed basic variables with standard normal distribution and their realizations
U_i = material constant in Ref. (8)
u^* = β point
V^*_1, V^*_2 = in-plane lamination parameter
var(X) = variance of X
v_i = ply ratio of the ith ply group
W^*_1, W^*_2 = flexural lamination parameter
X, X_i = random variable
x = axis along fibers
y = axis perpendicular to fiber direction
z = axis perpendicular to the laminate
z_i = distance between the midplane to the upper surface of the ith ply group of the laminate
β = safety index
ϵ = in-plane strain vector; strain
ϵ_{i0} = upper bound of strain, i = I, II, VI
Θ = vector of fiber orientation angles of a laminate
θ = fiber orientation angle
θ_i = fiber orientation angle of the ith ply group
κ = curvature vector
ν = Poisson's ratio
ρ_{ij} = correlation coefficient between θ_i and θ_j
$\sigma_{\theta i}$ = standard deviation of fiber orientation angle
Φ = standard normal distribution function

Subscripts

D = failure domain where $M \leq 0$
x, y = longitudinal and transversal
s = shear in x-y axes; symmetry of laminate
1, 2 = major and minor reference axes
6 = shear in 1-2 axes
I, II = major and minor principal strain
VI = maximum shear strain

Presented as Paper 92-2350 at the AIAA/ASME/ASCE/AHS/ASC 33rd Structures, Structural Dynamics, and Materials Conference, Dallas, TX, April 13–15, 1992; received Aug. 8, 1992; revision received April 2, 1993; accepted for publication April 2, 1993.

*Associate Professor, Department of Aerospace Engineering. Member AIAA.
†Professor, Department of Aerospace Engineering. Member AIAA.
‡Graduate Student, Department of Aerospace Engineering.
§Engineer, Product Planning Department.

Introduction

LAMINATED fibrous composite plates can be tailored to obtain the maximum performance by selecting ply materials and taking an optimum stacking sequence. Many investigations have been carried out for obtaining the optimum fiber angles for the maximum stiffness,[1] the maximum buckling strength,[2–4] and the maximum fundamental frequency.[5] In addition to those researches, an approach using the lamination parameters is one of the most powerful methods since it clearly gives the feasible design region for general stacking sequences for multiaxial angleply symmetric laminates and yields analytical solutions for many problems.[6-11]

Those studies, however, assume deterministic conditions where the material constants of fibrous laminated composites and the applied stresses have no variations. Miki et al.[12] developed a method to evaluate the reliability of the strength of unidirectional fibrous composites based on the advanced first-order second-moment (AFOSM) method and found that the optimum fiber angle which yields the maximum strength reliability changed with the increase in the variation of the applied load. Miki et al.[13] also proposed a new analytical method to determine the optimum fiber angle of unidirectional composites under probabilistic conditions, which is called the interior tangent ellipsoid (ITE) method. This method clearly shows that the optimum fiber angles are different between probabilistic and deterministic conditions. For the comparative verification with respect to various failure criteria, Nakayasu et al.[14] proposed a reliability model of unidirectional fibrous composites based on the first-/second-order reliability method. Cederbaum et al.[15] investigated many aspects regarding the reliability of composite structures under random vibration.

For multiaxial laminates, Cederbaum et al.[16] proposed a method for evaluating the reliabilities associated with various failure modes of each ply of laminates. Shao et al.[17] investigated the reliability-based optimum fiber orientation angles of multiaxial laminates and showed that the optimum laminate configurations with the maximum reliability of the strength under probabilistic conditions are very different from those with the maximum strength under deterministic conditions. However, the optimum laminate configuration for stiffness constraints under probabilistic conditions has not been studied as yet. The optimum laminate configurations for stiffness-related properties can change under probabilistic conditions where the material constants of fibrous laminated composites and the applied stresses have some variations.

In this study, a new approach is proposed to incorporate the variations of the elastic constants of composites and applied loads into optimum design with stiffness constraints, using the lamination parameters. The key idea is the combination of reliability analysis and the lamination parameter method.

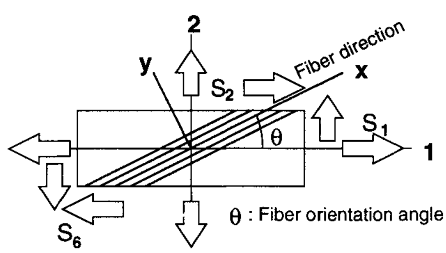

Fig. 1 Coordinate systems.

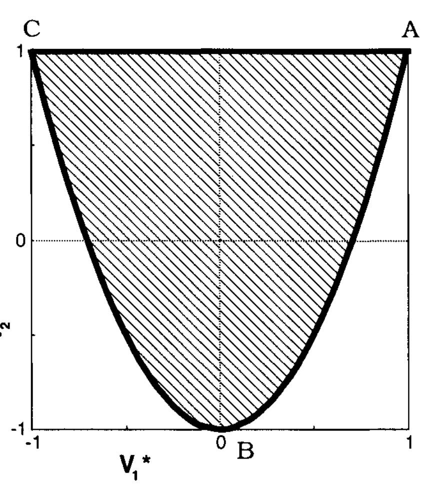

Fig. 2 Allowable region of ILP.

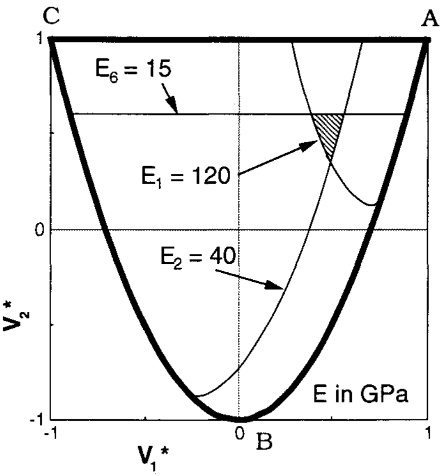

Fig. 3 Feasible design region (hatched area) for the constraints given by Eq. (10).

Laminate Design with Stiffness Constraints

The coordinate system for laminated plates is shown in Fig. 1. The relation between in-plane load N, moment M, in-plane strain ϵ, and curvature κ of a symmetric laminate is represented as

$$N = A\epsilon, \qquad M = D\kappa$$
$$A = hA^*, \qquad D = h^3D^*/12 \tag{1}$$

For the laminates having orthotropic lamina, the elements of normalized in-plane stiffness matrix A^* and normalized flexural stiffness matrix D^* are given by[18]

$$\begin{Bmatrix} A_{11}^* \\ A_{22}^* \\ A_{12}^* \\ A_{66}^* \end{Bmatrix} = \begin{bmatrix} U_1 & V_1^* & V_2^* \\ U_1 & -V_1^* & V_2^* \\ U_4 & 0 & -V_2^* \\ U_5 & 0 & -V_2^* \end{bmatrix} \begin{Bmatrix} 1 \\ U_2 \\ U_3 \end{Bmatrix} \tag{2}$$

$$\begin{Bmatrix} D_{11}^* \\ D_{22}^* \\ D_{12}^* \\ D_{66}^* \end{Bmatrix} = \begin{bmatrix} U_1 & W_1^* & W_2^* \\ U_1 & -W_1^* & W_2^* \\ U_4 & 0 & -W_2^* \\ U_5 & 0 & -W_2^* \end{bmatrix} \begin{Bmatrix} 1 \\ U_2 \\ U_3 \end{Bmatrix} \tag{3}$$

where

$$V_1^* = \frac{2}{h}\int_0^{h/2} \cos 2\theta \, dz, \qquad V_2^* = \frac{2}{h}\int_0^{h/2} \cos 4\theta \, dz \tag{4}$$

$$W_1^* = \frac{24}{h^3}\int_0^{h/2} \cos 2\theta z^2 \, dz, \qquad W_2^* = \frac{24}{h^3}\int_0^{h/2} \cos 4\theta z^2 \, dz \tag{5}$$

The stacking sequence of the laminate of which each lamina is orthotropic is represented by

$$[(\pm\theta_n)_{Nn}/\cdots/(\pm\theta_2)_{N2}/(\pm\theta_1)_{N1}]_s \tag{6}$$

A typical design problem for optimizing laminates with stiffness constraints is formulated as

$$\text{optimize } f(\Theta, h) \text{ subject to } g_k(A^*, D^*, h) \le 0$$
$$(k = 1, 2, \ldots, K) \tag{7}$$

where the function f can be any function of the stacking sequence and the thickness of a laminate.

This type of optimization problem is generally solved by using a nonlinear mathematical programming method. Hirano,[2] for example, obtained the optimum fiber orientations for the maximum buckling strength by using Powell's method. However, such an approach is not efficient for problems having a large number of design variables since there are a lot of local optima. Therefore, a more general method is needed.

Laminate Design Using Lamination Parameters

The design method using the lamination parameters is one of such general methods. Miki and Sugiyama[11] proposed a unified approach to the design of laminates using the lamination parameters. The lamination parameters are V^* and W^* in Eqs. (4) and (5), and they are called the in-plane lamination parameters (ILP) and the flexural lamination parameters (FLP), respectively. Miki[6,7] investigated the allowable region of the lamination parameters and found that the parameters play an important role in laminate design.

Design problems under in-plane loads are considered here. The allowable region of ILP is obtained as[6,11]

$$V_2^* \geq V_1^{*2} - 1, \qquad V_2^* \leq 1 \tag{8}$$

which is represented as the hatched region in Fig. 2. Then the general description of the problem is stated as

$$\text{optimize}\, f(V_1^*, V_2^*, h) \text{ subject to } g_k(A^*, h) \leq 0$$
$$(k = 1, 2, \ldots, K) \tag{9}$$

Many in-plane stiffness-related functions are evaluated on the allowable region, and a feasible design region can be easily obtained. For example, in-plane effective engineering constants are the functions of in-plane stiffness which is the function of ILP. Therefore, the engineering constants can be evaluated on the ILP plane.

Suppose that the following design constraints are given for in-plane effective engineering constants:

$$E_1 \geq 120, \qquad E_2 \geq 40, \qquad E_6 \geq 15 \text{ GPa} \tag{10}$$

The hatched region in Fig. 3 is the feasible design region for the given constraints for T300/5208 graphite/epoxy of which material constants are shown in Table 1. A designer selects a ply material and chooses one design point in the region, and then the values of the lamination parameters are calculated. From these values, the stacking sequence is obtained by using Eq. (4) after giving the number of different orientation angles and the number of plies.

If a designer chooses an angleply symmetric laminate with five axes, the laminate configuration is $[(\pm\theta_1)_{p1}, (\pm\theta_2)_{p2}, (0)_{p3}]_s$, which is called penta-axial laminates here. This type of laminate is reduced to a tetra-axial laminate as p_3 becomes zero and to a triaxial laminate as p_1 or p_2 becomes zero. It should be noted that the representation of laminate configuration gives only the fiber orientation angles and their ply ratios and that it does not yield a specific stacking sequence since the in-plane stiffness is not affected by stacking sequence. Equation (4) is rewritten for this type of laminate as

$$V_1^* = p_1\cos 2\theta_1 + p_2\cos 2\theta_2 + p_3$$
$$V_2^* = p_1\cos 4\theta_1 + p_2\cos 4\theta_2 + p_3 \tag{11}$$

where

$$\Sigma p_i = 1 \qquad (i = 1, 2, 3) \tag{12}$$

The fiber orientation angles in Eq. (11) are calculated from the values of ILP of the design point, but the laminate configuration is not uniquely obtained since the number of the design variables is four. In this case, two of the ply ratios are predetermined by considering the total number of plies of the laminate, and the remaining variables are calculated. Those solutions are equivalent with respect to the in-plane stiffness.

The strains of a composite plate are evaluated on the ILP plane when applied stresses are given. Therefore, strain-related performance such as a strain-based failure criterion can be evaluated on the ILP plane. Then the lamination parameter method can be used for strength problems as well as stiffness problems.

Probabilistic Stiffness Design of Laminates

The material constants and the applied loads in the stiffness design are assumed to be deterministic in the previous section. However, they have some uncertainties in practice, and so stiffness design problems under probabilistic conditions are discussed in the following part of the present paper. The concept of structural reliability is introduced to solve such problems.

The problem given in Eq. (9) is rewritten under probabilistic conditions as

$$\text{optimize}\, f(V_1^*, V_2^*, h) \text{ subject to } P[g_k(A^*, h) \leq 0] \geq P_0$$
$$(k = 1, 2, \ldots, K) \tag{13}$$

where P_0 is the prescribed lower bound of probability. This means that the constraints are satisfied at the probability level higher than P_0. The material constants and the applied stresses are treated as random variables. The variations in fiber orientation angles are not considered here.

To solve this problem, it is necessary to find the feasible design region with respect to the probabilistic constraints in Eq. (13), and a design point has to be determined. The laminate configuration is obtained in the same way as mentioned in the previous section after obtaining the design point on the ILP plane.

Evaluation of Probability on In-Plane Lamination Parameters Plane

In structural reliability analysis, a limit state function which defines failure of a structure is introduced. The limit state function is represented as a function of variables X with some uncertainties called basic random variables that affect the failure as

$$M(x) = g(X_1, X_2, \ldots, X_n) \tag{14}$$

where $M \leq 0$ means failure, and $M > 0$ means nonfailure. A plane which satisfies $M = 0$ is called a limit state or a failure surface. The basic random variables are assumed to be statistically independent or to have been transformed into independent variables.

After defining the limit state function, the probability of failure of a structure is represented as

$$P_f = \iint \cdots \int_D f_x(X_1, X_2, \ldots, X_n)\, dX_1 \cdot dX_2 \cdots dX_n \tag{15}$$

where D is a failure domain satisfying $M \leq 0$.

The evaluation of the probability is performed by using the AFOSM method[20-22] where the probability of failure is evaluated by the reliability index β that is a function of the first and second moments (mean and variance) of the linear approximation of the limit state function.

Table 1 Material constants for the analysis

	E_x, GPa	E_y, GPa	E_s, GPa	ν_x
Mean[a]	181	10.3	7.17	0.28
Coeff. of variation[b]	0.05	0.05	0.05	0.01
Distribution[b]		Normal		

[a]Mean values are from Ref. 19. [b]Coefficients of variation and the distributions are assumed.

In the AFOSM method, the basic random variables X are transformed into standard normal variables, U.

$$U_i = \Phi^{-1}[F_{x_i}(X_i)] \tag{16}$$

The linearization of the limit state function is carried out at such a point u^* that yields the shortest distance between the point on the failure surface and the origin in the U space. Point u^* is called the β point, and the reliability index β is given as the distance between the origin and point u^* as shown in Fig. 4.

The β point is usually obtained by using an iterative method[20-23] or a nonlinear mathematical programming method.[24] This method is called the AFOSM method. The probability of failure, that is, the probability of event $M \le 0$, is obtained from β as

$$P_f = \Phi(-\beta) \tag{17}$$

The evaluation of the reliability of the stiffness of the laminates on the lamination parameter plane is performed as follows. The limit state function for this problem is considered to be $g_k(A^*, h)$ in Eq. (13), and the probability $P[g_k(A^*, h) < 0]$ can be evaluated by using the reliability index β after giving IPL. A new algorithm by Shao et al.[17] is used in this paper.

The effective longitudinal modulus along the 1 axis, E_1, for example, is the function of the in-plane stiffness A^*, and its contour curve with a constant reliability level is drawn on the IPL plane as shown in Fig. 5. The material constants are assumed to be normally distributed with their mean values listed in Table 1, and their coefficients of variation are given in the figure. This figure also shows the effect of the coefficients of variation of the material constants of the ply. The domain providing the reliability of more than 0.95 is the upper-right portion of the figure, and it is reduced as the variation in the material constants increases.

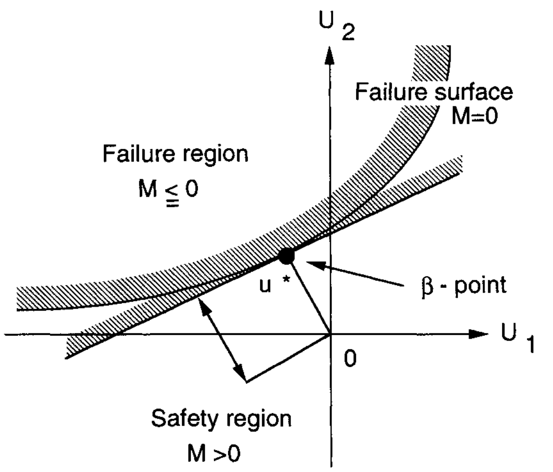

Fig. 4 β point in U space.

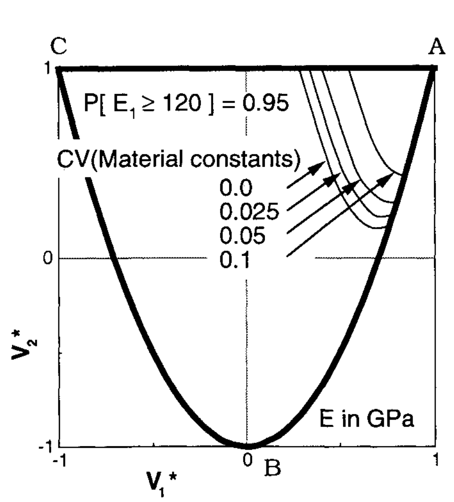

Fig. 5 Contour curves of E_1 on the ILP plane.

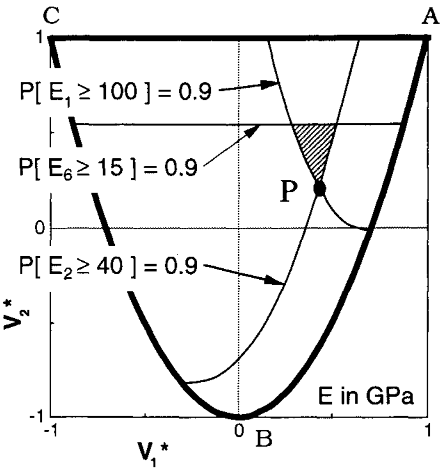

Fig. 6 Feasible design region for problem A.

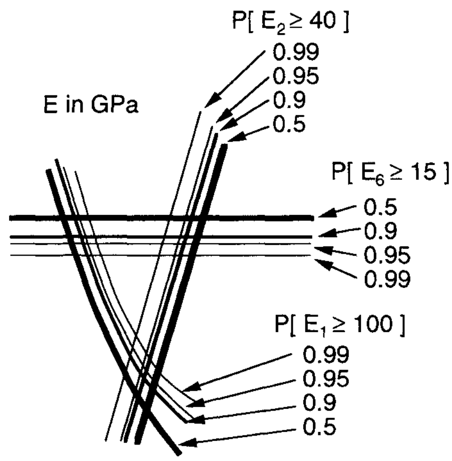

Fig. 7 Detail around the feasible design region.

This is different from deterministic cases where the contour curve remains unchanged. In this case, the contour curve for $E_1 = 120$ GPa under deterministic conditions corresponds to the curve for $CV = 0$ in Fig. 5.

Results and Discussions

Typical Design Problems

Three types of design problems are considered here.

Problem A

Problem A is an optimization problem where the objective function is a function of the in-plane stiffness of a laminate such as E_i or A_{ij}, and the constraints are imposed on the effective engineering constants of a laminate. Effective shear constant E_6 is adopted here as the objective function, for simplicity.

$$\text{Maximize } E_6 \text{ subject to } P[E_i \ge E_{i0}] \ge P_0 \; (i = 1, 2, 6) \tag{18}$$

where $E_{10} = 100$, $E_{20} = 40$, $E_{60} = 15$ (GPa); $P_0 = 0.5, 0.9, 0.95, 0.99$.

Problem B

Problem B is a problem where the constraints are imposed on the principal strains of a laminate under given applied loads. It is very important to investigate the maximum strain

Table 2 Laminate configurations for problem A; laminate type: $[(\pm\theta_1)p_1, (\pm\theta_2)p_2, (0)p_3]_s$

No.	θ_1, deg	p_1	θ_2, deg	p_2	θ_3, deg	p_3
01	10.3	0.6	56.7	0.4	0	0.0
02	16.9	0.7	63.2	0.3	0	0.0
03	21.6	0.8	77.2	0.2	0	0.0
04	11.3	0.5	56.7	0.4	0	0.1
05	18.4	0.6	63.1	0.3	0	0.1
06	23.3	0.7	76.4	0.2	0	0.1
07	12.7	0.4	56.6	0.4	0	0.2
08	20.4	0.5	62.9	0.3	0	0.2
09	25.5	0.6	75.3	0.2	0	0.2
10	14.8	0.3	56.6	0.4	0	0.3
11	23.3	0.4	62.5	0.3	0	0.3
12	28.6	0.5	73.6	0.2	0	0.3
13	18.5	0.2	56.5	0.4	0	0.4
14	27.8	0.3	61.9	0.3	0	0.4
15	33.2	0.4	71.0	0.2	0	0.4
16	28.2	0.1	56.2	0.4	0	0.5
17	37.4	0.2	60.0	0.3	0	0.5
18	41.8	0.3	65.3	0.2	0	0.5
19[a]	45.2	0.4	80.0	0.1	0	0.5

[a]Total number of plies = 20.

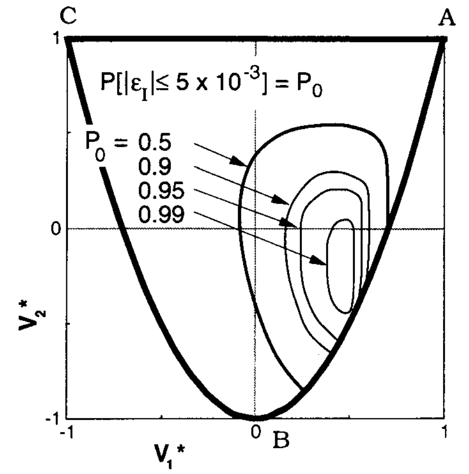

Fig. 8 Feasible region for the constraint of the major principal strain.

of a laminate since it is needed for the maximum strain criterion of strength. The probability levels are set to several values. The probability of 0.5 corresponds to the design based on mean values. Find the feasible design region for the following constraints:

$$\text{constraints}: P[|\epsilon_i| \le \epsilon_{i0}] \ge P_0 \; (i = \text{I, II, VI}) \quad (19)$$

where $\epsilon_{\text{I}0} = 5 \times 10^{-3}$, $\epsilon_{\text{II}0} = 8 \times 10^{-3}$, $\epsilon_{\text{VI}0} = 9.5 \times 10^{-3}$; $P_0 = 0.5, 0.9, 0.95, 0.99$; $E(S_1) = 0.3$, $E(S_2) = 0.1$, $E(S_6) = 0.1$ (GPa); and $CV(S_i) = 0.2$ ($i = 1, 2, 6$).

Problem C

Problem C is an optimization problem where the constraint is imposed on an approximate failure criterion[18] of a laminate which is expressed in terms of the laminate strains. The loading condition is the same as that in problem B. The transverse strain is adopted as the objective function, for simplicity.

$$\text{Minimize } \epsilon_2 \text{ subject to } P[\epsilon_1^2 + \epsilon_2^2 + 0.5\,\epsilon_6^2 \le K^2] \ge P_0 \quad (20)$$

where $K = 5 \times 10^{-3}$; $P_0 = 0.5, 0.9, 0.95, 0.99$.

Feasible Design Region and Design Point

The feasible design region of problem A is shown in Fig. 6 with the probability level of 90% and the material properties listed in Table 1. The detail of the feasible design region for various probability levels is illustrated in Fig. 7. It is clearly recognized that the feasible design region shrinks with increasing the probability level. If there is no feasible design region, the ply material should be changed to one with higher properties.

The objective function in problem A is the effective shear modulus E_6 and the optimum design point becomes point P in Fig. 6 since E_6 increases with the decrease in V_2^*. The values of the lamination parameters of point P are

$$V_1^* = 0.403, \qquad V_2^* = 0.177 \quad (21)$$

The laminate configuration is obtained from Eq. (11). The calculated result is shown in Table 2 where the total number of plies is assumed to be 20, and then the ply ratio is a multiple of 0.1. All configurations in Table 2 have equal in-plane stiffness, and a designer can choose one configuration based on some other criteria.

For problem B, the feasible regions for each constraint are shown in separate figures. Figures 8, 9, and 10 show the feasible regions for the major principal strain, the minor principal strain, and the maximum shear strain, respectively. The feasible regions are reduced with increasing the probability level. It should be noted that these regions are affected by the applied stresses. The applied stresses are assumed to be independent Gaussian random variables with their mean values of $E(S_1) = 0.3$, $E(S_2) = 0.1$, and $E(S_6) = 0.1$ (GPa), and their coefficients of variation $CV(S_i) = 0.2$ ($i = 1, 2, 6$). The material constants are assumed to have the probabilistic properties given in Table 1. The major principal strain is found to be critical from these results.

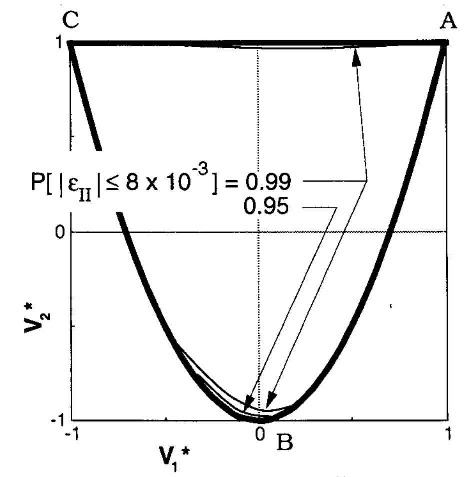

Fig. 9 Feasible region for the constraint of the minor principal strain.

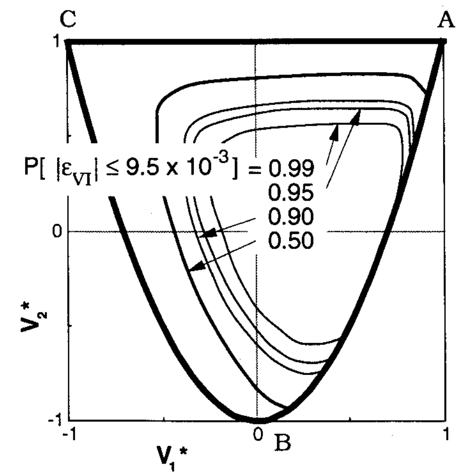

Fig. 10 Feasible region for the constraint of the maximum shear strain.

The overall feasible design region for problem B is shown in Fig. 11, which is very similar to the feasible region for the major principal strain although it is slightly different by the effect of the constraint on the maximum shear strain. This region corresponds to nonfailure region on the maximum strain criterion, and it can be seen that probabilistic strength constraints are evaluated on the lamination parameter plane.

The feasible design region for problem C is shown in Fig. 12. The numerical data and probabilistic properties of the material constants and the applied stresses are the same as those for problem B. It is clearly recognized that this result is very similar to Fig. 11, and it is found that the approximate failure criterion in terms of laminate strains is effective. When the probability level is set to 95% and the objective is to minimize the transverse strain, then the optimum design point becomes point P in the figure. The values of the lamination parameters are

$$V_1^* = 0.340, \qquad V_2^* = -0.390 \tag{22}$$

The laminate configurations are obtained from Eq. (11). The laminate configurations are listed in Table 3 where the total number of plies is assumed to be 20.

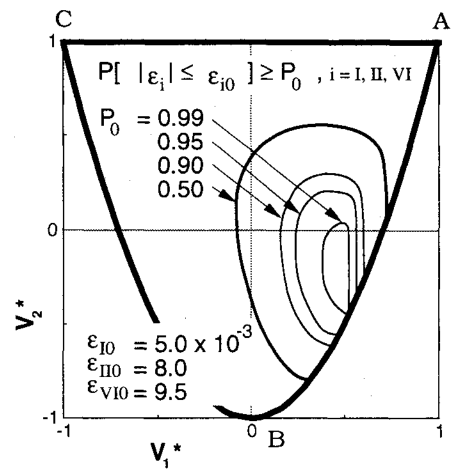

Fig. 11 Feasible design region for problem B.

The laminate configurations in Tables 2 and 3 are equivalent with respect to their in-plane stiffnesses, respectively. Therefore, a designer can choose one configuration after evaluating other properties of the laminates.

Minimization of the Variation in In-Plane Lamination Parameters

One method for selecting a laminate configuration among the equivalent ones with respect to their in-plane stiffness is to minimize the variation in the lamination parameters caused by the variation in the fiber orientation angles. The purpose is to make the actual design point as close as possible to the theoretical design point on the ILP plane.

The sum of the variances of ILPs, SV, is approximated as

$$\begin{aligned} SV &= \mathrm{var}(V_1^*) + \mathrm{var}(V_2^*) \\ &= 4\frac{N_1}{N^2}(1-t_1^2)(1+16t_1^2)\sigma_{\theta 1}^2 + 4\frac{N_2}{N^2}(1-t_2^2)(1+16t_2^2)\sigma_{\theta 2}^2 \\ &+ 4\frac{N_1(N_1-1)}{N^2}(1-t_1^2)(1+16t_1^2)\rho_{11}\sigma_{\theta 1}^2 \\ &+ 4\frac{N_2(N_2-1)}{N^2}(1-t_2^2)(1+16t_2^2)\rho_{22}\sigma_{\theta 2}^2 \\ &+ 8\frac{N_1N_2}{N^2}\sqrt{(1-t_1^2)(1-t_2^2)}(1+16t_1t_2)\rho_{12}\sigma_{\theta 1}\sigma_{\theta 2} \end{aligned} \tag{23}$$

where $t_i = \cos 2\theta_i (i = 1, 2)$.

The sum of these two variances is adopted as the evaluation function for selecting an optimum configuration from the equivalent solutions with respect to their in-plane stiffness. For the configurations in Table 2, the comparison of the evaluation functions is shown in Fig. 13 where the standard deviations of the orientation angles are assumed to be 5 deg, and the correlation coefficients ρ_{ij} between the fiber angles of each ply are assumed to be 0 or 1. Configuration 19 is found to be optimum for both cases.

When the orientation angles are assumed to be independent Gaussian random variables and the other random variables are taken to be the same as those for problem C, the probabilities of satisfying the constraint Eq. (20) are evaluated for the configurations in Table 3 by using the AFOSM method and

Table 3 Laminate configurations for problem C; laminate type: $[(\pm\theta_1)p_1, (\pm\theta_2)p_2, (0)p_3]_s$[a]

								$P[\epsilon_1^2+\epsilon_2^2+0.5\epsilon_6^2 \le K^2]$	
No.	θ_1, deg	p_1	θ_2, deg	p_2	θ_3, deg	p_3	$SV(\rho_{ij}=0)$	AFOSM	MC[b]
01	14.6	0.4	45.4	0.6	0	0.0	0.566×10^{-2}	0.903	0.918
02	19.6	0.5	47.7	0.5	0	0.0	0.818	0.891	0.902
03	23.0	0.6	50.6	0.4	0	0.0	1.012	0.875	0.884
04	25.7	0.7	54.5	0.3	0	0.0	1.162	0.859	0.867
05	28.1	0.8	61.0	0.2	0	0.0	1.244	0.843	0.852
06	30.5	0.9	82.4	0.1	0	0.0	1.032	0.825	0.842
07	17.1	0.3	45.4	0.6	0	0.1	0.529	0.911	0.925
08	22.4	0.4	47.5	0.5	0	0.1	0.718	0.903	0.916
09	25.8	0.5	50.1	0.4	0	0.1	0.847	0.892	0.903
10	28.4	0.6	53.5	0.3	0	0.1	0.939	0.881	0.890
11	30.7	0.7	59.1	0.2	0	0.1	0.983	0.870	0.878
12	33.0	0.8	74.2	0.1	0	0.1	0.846	0.858	0.870
13	21.8	0.2	45.2	0.6	0	0.2	0.455	0.920	0.931
14	27.1	0.3	47.0	0.5	0	0.2	0.553	0.915	0.928
15	30.3	0.4	49.1	0.4	0	0.2	0.608	0.910	0.922
16	32.6	0.5	51.8	0.3	0	0.2	0.642	0.903	0.914
17	34.5	0.6	56.0	0.2	0	0.2	0.656	0.896	0.907
18	36.4	0.7	65.8	0.1	0	0.2	0.603	0.890	0.900
19	39.0	0.1	44.1	0.6	0	0.3	0.235	0.924	0.934
20	40.6	0.2	44.5	0.5	0	0.3	0.235	0.929	0.937
21	41.3	0.3	44.9	0.4	0	0.3	0.235	0.929	0.938
22	41.8	0.4	45.4	0.3	0	0.3	0.235	0.928	0.937
23	42.2	0.5	46.1	0.2	0	0.3	0.235	0.924	0.935
24	42.6	0.6	47.7	0.1	0	0.3	0.236	0.919	0.931

[a]Total number of plies = 20. [b]Monte Carlo simulation, sample size = 10,000.

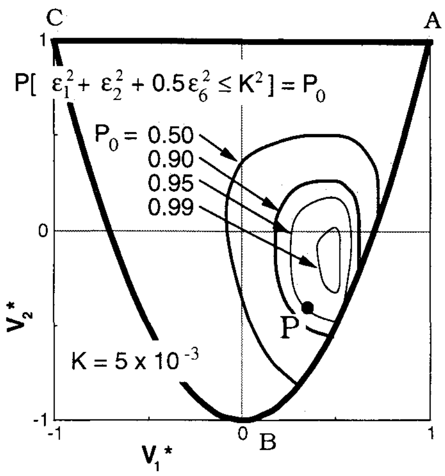

Fig. 12 Feasible design region for problem C.

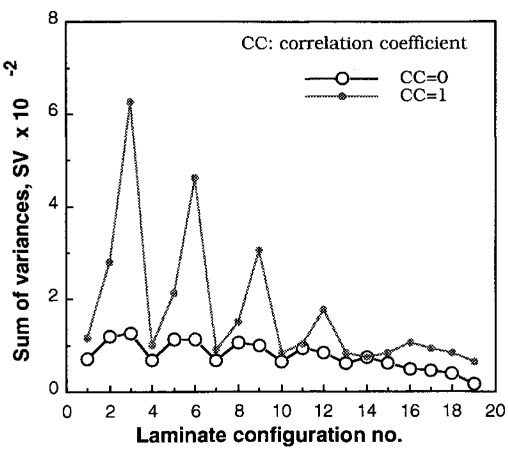

Fig. 13 Variation in ILP.

Monte Carlo simulation. The results are also listed in Table 3. It should be remarked here that the probability level has been set to 0.95 where no uncertainties exist in the orientation angles. It is seen that the probability levels become lower than the prescribed value 0.95 when the orientation angles have probabilistic variations. It is observed that the sum of the variances of ILPs is closely related to the probability levels of the constraint, that is, the small sum of the variances results in the high probability of satisfying the constraint. This confirms that the sum of the variances of ILPs can be used as the performance index for selecting the optimum configuration from the equivalent solutions.

From this result, the following points are observed. Firstly, the variation in ILP is larger for the large value of correlation coefficient. Secondly, the increase in the ply ratio of 0 deg yields the decrease in the variation. This result depends on the values of ILP, but it can be stated that if the constraint along the 1 axis is dominant the use of 0-deg ply as much as possible provides the lowest variation in ILP. The variation in ILP becomes important when the fiber orientation angles of the plies in laminate have some variation.

Conclusions

A reliability analysis is introduced into the lamination parameter method, and a new approach to determine optimum laminate configurations in optimum design problems with probabilistic constraints on the stiffness or stiffness-related properties of laminated fibrous composites is proposed. The following conclusions are drawn from this study.

The formulation for the reliability analysis of the stiffness-related properties of laminated composites is presented. A feasible design region associated with probabilistic constraints on the stiffness or stiffness-related properties of composite laminates can be obtained on the lamination parameter plane. The feasible region is reduced with the increase in the variation in the material constants of a ply material. The feasible design region is reduced with the increase in the probability of satisfying given constraints.

Some optimum laminate configurations which are equivalent with respect to their stiffness are obtained from the optimum lamination parameters. A method is proposed to determine one configuration among those equivalent ones to minimize the variation in lamination parameters, that is, the variation in the stiffness properties for the case where some variations in the fiber orientation angles exist.

References

[1]Tauchert, T. R., and Adibhatla, S., "Design of Laminated Plates for Maximum Stiffness," *Journal of Composite Materials,* Vol. 18, Jan. 1984, pp. 58–69.

[2]Hirano, Y., "Optimum Design of Laminated Plates under Axial Compression," *AIAA Journal,* Vol. 17, No. 7, 1979, pp. 1017–1019.

[3]Hirano, Y., "Optimum Design of Laminated Plates Under Shear," *Journal of Composite Materials,* Vol. 13, Oct. 1979, pp. 329–335.

[4]Starnes, J. H., Jr., and Haftka, R. T., "Preliminary Design of Composite Wings for Buckling, Strength, and Displacement Constraints," *Journal of Aircraft,* Vol. 16, No. 8, 1979, pp. 564–570.

[5]Reiss, R., and Ramachandran, S., "Maximum Frequency Design of Symmetric Angle-ply Laminates," *Composite Structures 4,* Vol. 1, 1987, pp. 1.476–1.487.

[6]Miki, M., "Material Design of Composite Laminates with Required In-Plane Elastic Properties," *Progress in Science and Engineering of Composites, Proceedings of the 4th International Conference on Composite Materials,* Vol. 2, Japan Society for Composite Materials, Tokyo, Japan, 1982, pp. 1725–1731.

[7]Miki, M., "Design of Laminated Fibrous Composite Plates with Required Flexural Stiffness," *Recent Advances in Composites in the United States and Japan,* ASTM STP 864, edited by J. R. Vinson and M. Taya, American Society for Testing and Materials, Philadelphia, PA, 1985, pp. 387–400.

[8]Miki, M., "Optimum Design of Fibrous Laminated Composite Plates Subject to Axial Compression," *Composites '86: Recent Advances in Japan and the United States,* edited by K. Kawata, S. Umekawa, and A. Kobayashi, Proceedings of the 3rd Japan-U.S. Conference on Composite Materials, Japan Society for Composite Materials, Tokyo, Japan, 1986, pp. 673–680.

[9]Grenestedt, J. L., "Layup Optimization and Sensitivity Analysis of the Fundamental Eigenfrequency of Composite Plates," *Composite Structures,* Vol. 12, 1989, pp. 193–209.

[10]Fukunaga, H., "Stiffness and/or Strength Optimization of Laminated Composites," *Composites '86: Recent Advances in Japan and the United States,* edited by K. Kawata, S. Umekawa, and A. Kobayashi, Proceedings of the 3rd Japan-U.S. Conference on Composite Materials, Japan Society for Composite Materials, Tokyo, Japan, 1986, pp. 655–662.

[11]Miki, M., and Sugiyama, Y., "Optimum Design of Laminated Composites Plates Using Lamination Parameters," *Proceedings of the AIAA/ASME/ASCE/AHS/ASC 32nd Structures, Structural Dynamics, and Materials Conference,* AIAA, Washington, DC, 1991, pp. 275–283 (AIAA Paper 91-0971).

[12]Miki, M., Murotsu, Y., Tanaka, T., and Shao, S., "Reliability of the Strength of Unidirectional Fibrous Composites," *AIAA Journal,* Vol. 28, No. 11, 1990, pp. 1980–1986.

[13]Miki, M., Murotsu, Y., and Tanaka, T., "Optimum Fiber Angle of Unidirectional Composites for Load with Variations," *Proceedings of the AIAA/ASME/ASCE/AHS/ASC 31st Structures, Structural Dynamics, and Materials Conference,* AIAA, Washington, DC, 1990, pp. 1333–1339 (AIAA Paper 90-1071).

[14]Nakayasu, H., Maekawa, Z., and Rackwitz, R., "Reliability-Oriented Materials Design of Composite Materials," *Proceedings of ICOSSAR '89, The 5th International Conference on Structural Safety and Reliability,* 1990, pp. 2095–2098.

[15]Cederbaum, G., Elishakoff, I., Aboudi, J., and Librescu, L., *Random Vibrations and Reliability of Composite Structures,* Technomic, Lancaster, PA, 1992.

[16]Cederbaum, G., Elishakoff, I., and Librescu, L., "Reliability of Laminated Plates via the First-Order Second Moment Method," *Composite Structures,* Vol. 15, 1990, pp. 161–167.

[17]Shao, S., Miki, M., and Murotsu, Y., "Optimum Fiber Orientation Angle of Multiaxially Laminated Composites Based on Reliability," *Proceedings of the AIAA/ASME/ASCE/AHS/ASC 32nd Structures, Structural Dynamics, and Materials Conference,* AIAA, Washington, DC, 1991, pp. 1280–1287 (AIAA Paper 91-1032).

[18]Tsai, W., *Composite Design,* 3rd ed., Think Composites, Dayton, OH, 1987, Sec. 7, 8.

[19]Tsai, S. W., and Hahn, H. T., *Introduction to Composite Materials,* Technomic, Lancaster, PA, 1980, p. 19.

[20]Thoft-Christensen, P., and Murotsu, Y., *Application of Structural Systems Reliability Theory,* Springer Verlag, Berlin, Germany, 1986, pp. 15–24.

[21]Madsen, H. O., Krenk, S., and Lind, N. C., *Methods of Structural Safety,* Prentice Hall, Englewood Cliffs, NJ, 1986, p. 44.

[22]Hasofer, A. M., and Lind, N. C., "Exact and Invariant Second Moment Code Format," *Journal of the Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers,* Vol. 100, No. EM1, 1974, pp. 111–121.

[23]Rackwitz, R., and Fiessler, B., "Structural Reliability under Combined Random Sequences," *Computers and Structures,* Vol. 9, 1978, pp. 489–494.

[24]Shinozuka, M., "Basic Analysis of Structural Safety," *Journal of the Structural Division, American Society of Civil Engineers,* Vol. 109, No. 3, 1983, pp. 721–740.